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STRESSES, VISCOSITY, AND SCALES IN MOLAR TRANSFER

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UDC 532.526

The article presents unified relations describing the profiles of molar stresses and viscosity.

For the calculation of heat and mass transfer in engineering devices it is indispensable to know the distribution of molar stresses and viscosity in the turbulent boundary layer. However, to this day there is no unified relations describing the molar stress and viscosity profiles for complex flow conditions, e.g., in case of a rough surface, the entrance section of a channel, etc. [1].

It is known [1] that in the turbulent core of a two-dimensional steady turbulent boundary layer the terms of the stress tensor satisfy the inequality

$$\sigma_u^{+2} > \sigma_w^{+2} > \sigma_v^{+2} > \tau^+. \tag{1}$$

For flow in the boundary layer on a plate, taking (1) into account, we represent σ_v^{+2} in the form

$$\tau^+ = \sigma_v^{+2} - \xi_v \sigma_v^{+2}. \tag{2}$$

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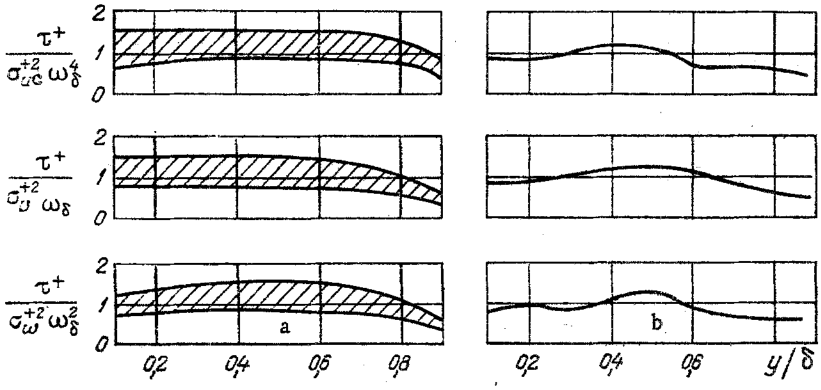

Fig. 1. Molar stress profiles in flow: a) in rectangular channels and pipes with smooth walls after [10-15]; b) in a rough pipe after [12]. ($k^+ = 68.5$; $\Psi_\delta = 0.46$).

Here ξ_v is a damping factor which reduces σ_v^{+2} to a value at which (2) is fulfilled. Analogously

$$\sigma_v^{+2} = \sigma_w^{+2} - \xi_w \sigma_w^{+2}, \tag{3}$$

$$\sigma_w^{+2} = \sigma_u^{+2} - \xi_u \sigma_u^{+2}. \tag{4}$$

In distinction to flow on a plate, turbulent flow in a channel (pipe) is characterized by interaction of the vortices forming at opposite walls. These vortices have predominantly opposite spins and are prone to merging. In case of such additional interaction of vortices there are no grounds for assuming that expressions (2), (3) will change. As regards the rms pulsation of longitudinal speed in the channel σ_{uc}, we determine it in the following manner:

$$\sigma_u^{+2} = \sigma_{uc}^{+2} - \xi_{uc} \sigma_{uc}^{+2}. \tag{5}$$

In the general case $\xi_i = \xi_i(K)$, where K is a parameter characterizing the change of properties of the turbulent boundary layer when the flow conditions become more complex. It is obvious that the left-hand and right-hand sides of equalities (2)-(5) are equal to each other with an accuracy up to the unknown functions $\xi_v(K)$, $\xi_w(K)$, $\xi_u(K)$, $\xi_{uc}(K)$. It follows from the assumption that relations (2)-(5) are universal that ξ_i also have to be universal functions of the turbulent boundary layer.

In the model of molar transfer [2-9] the function Ψ_δ was obtained which determines the principal properties of the turbulent core of the boundary layer: $\Psi_\delta = \ln\delta^+/(u_\delta^+ - 1)$. Here $y^+ = yu_*/\nu$, $u_\delta = u(y = \delta)$. Therefore in the first approximation.

$$\xi_v = \xi_w = \xi_u = \xi_{uc} = \Psi_\delta. \tag{6}$$

Then

$$\sigma_v^{+2} = \tau^+/\omega_\delta, \tag{7}$$

$$\sigma_w^{+2} = \tau^+/\omega_\delta^2, \tag{8}$$

$$\sigma_u^{+2} = \tau^+/\omega_\delta^3, \tag{9}$$

$$\sigma_{uc}^{+2} = \tau^+/\omega_\delta^4, \tag{10}$$

where $\omega_\delta = 1 - \Psi_\delta$.

Expressions (7)-(9) apply to external turbulent flow (flow on a plate), expressions (7), (8), and (10) apply to internal flow (flow in a pipe or channel).

In Fig. 1a, expressions (7), (8), and (10) are compared with the data of [10-15] for turbulent flow in a rectangular channel or pipe with smooth walls ($Re_\delta = (15-125)\cdot 10^3$, $\Psi_\delta = 0.29-0.33$). To make the comparison more convenient, the ordinate in Fig. 1a is represented in the form of ratios of the right-hand sides of formulas (7), (8), and (10) to the left-hand sides. When the value of the ordinate is equal to unity, the experimental data coincide with the dependences given above, and when the value of the ordinate is, e.g., 1.5, the relative error amounts to 50%. The hatched areas correspond to the scatter of the experimental data [10-15] in such representation. The figure shows that there is satisfactory correlation between the experimental data and the theoretical dependences (7), (8), and (10) in the range $0.1 \leqslant y/\delta \leqslant 0.8$.

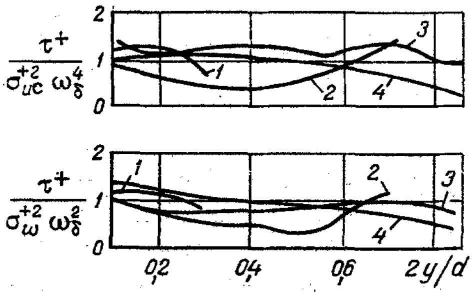

Fig. 2

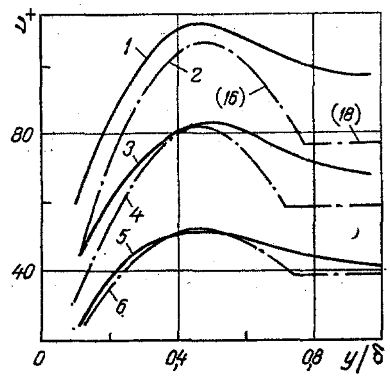

Fig. 3

Fig. 2. Molar stress profiles in flow in the entrance section of a smooth pipe after [17]: 1) x/d = 4.5; 2) 16.5; 3) 28.5; 4) 40.5.

Fig. 3. Molar viscosity profiles in flow in a smooth channel after [10]: 1, 3, 5) after [10]; 2, 4, 6) according to (16), and (18); 1, 2) Re = 32,300; 3, 4,) 23,200; 5, 6) 13,800.

In models of the transfer of kinetic energy, the following relation [16] is used for closing the differential equations:

$$\tau^+ = Ce^+. \tag{11}$$

Here $e^+ = e/u_*^2$; $e = \frac{1}{2}(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ is the kinetic energy of the turbulent motion per unit mass.

In accordance with [16], the values of the "constant" C in (11) lie most probably within the range 0.25-0.35, although the experimental data of [16] may indicate that in a turbulent boundary layer C may assume values from 0.1 to 0.5. The authors of [16] believe that in the boundary layer C is "slightly smaller than 0.3."

For external flow we have from (7)-(10) that

$$\frac{1}{C} = \frac{1}{2}\left(\frac{1}{\omega_\delta} + \frac{1}{\omega_\delta^2} + \frac{1}{\omega_\delta^3}\right), \tag{12}$$

for internal flow

$$\frac{1}{C_c} = \frac{1}{2}\left(\frac{1}{\omega_\delta} + \frac{1}{\omega_\delta^2} + \frac{1}{\omega_\delta^4}\right). \tag{13}$$

Since it is known [2] that on a smooth wall and for zero pressure gradient with $\delta^+ \to \infty$ $\Psi_\delta \to 1/3$, we obtain, when substituting this value into (12) and (13), that with turbulent flow on a plate C = 0.28, and in a channel C_c = 0.23.

A special feature of the experimental data presented in Fig. 1a is that an equal change of τ^+, characteristic of internal fully developed flow

$$\tau^+ = 1 - \frac{y}{\delta} \tag{14}$$

occurs, and the values of Ψ_δ are close.

When flow conditions are complex, the change of τ^+ in the boundary layer may differ from (14), and Ψ_δ may deviate from the degenerated value 1/3 with large δ^+. However, relations (7)-(10), and consequently also (12), (13), are conservative to such an effect.

It follows from [12] that with turbulent flow on a rough surface in a pipe, at least for the dimensionless height of roughness $k^+ \leqslant 100$, the above theoretical dependences are valid. In Fig. 1b relations (7), (8), and (10) are being compared with the experimental data of [12] (k^+ = 68.5, Ψ_δ = 0.46). Their satisfactory correlation can be seen. From (13) C_c = 0.12.

Let us examine an example where, with fixed value of Ψ_δ, the change of τ^+ in the layer differs substantially from (14). This is encountered, e.g., in the initial section of a channel. According to the data of [17], when x/d in a smooth pipe changes from 4.5 to 40.5,

Ψ_δ decreases from 0.35 to 0.31, i.e., the value of Ψ_δ is almost constant. Here, x is the distance from the entrance edge of the pipe, and d is its diameter. On the other hand, the distribution of τ over the section does not correspond to (14). For instance, with $x/d = 4.5$ and $2y/d = 0.2$, τ is approximately ten times smaller than its value for fully developed turbulent flow. However, in this case, too, the theoretical relations (7), (8), and (10) are satisfactorily correlated with the experimental data, as can be seen from Fig. 2.

On the basis of these examples we can convince ourselves that in (11) C is not a constant to approximately 0.3 but is determined by relations (12) or (13) in dependence on the type of turbulent flow: internal or external.

In accordance with the model of rolling of averaged vortices in the proper reference system [3], the expression for dimensionless turbulent viscosity has the form

$$\nu^+ = u_l^+ l^+ \tag{15}$$

or

$$\nu^+ = \Psi_\delta y^+ \tau^+, \tag{16}$$

where $\nu^+ = \nu_t/\nu$; $u_l^+ = u_l/u_*$; $l^+ = l u_*/\nu$; ν_t is turbulent viscosity; l is the radius of the averaged vortex; u_l is the speed of its rolling in the local base.

In expression (16) the fact is not taken into account that in the case of internal flow of a turbulent stream ν^+ at great distance from the wall tends to a constant value much larger than unity. This shortcoming of (16) can be eliminated if it is assumed that the quantization of the turbulent core according to averaged cores [3] has a definite physical meaning.

The difference between internal flow, for which $\nu^+ \to \text{const} \gg 1$ for $y \to \delta$, and external flow ($\nu^+ \to 0$ for $y \to \delta$) consists, as mentioned before, in the additional interaction, chiefly by merger of the vortices forming on opposite surfaces. The physical condition of the interaction of such averaged vortices can be reduced to the geometric condition of their contiguity because vortices merge only if they are directly contiguous. The condition of contiguity of averaged vortices, described by the superposition of [3] and rolling over a moving surface that is at the distance y_m^+ from the wall, has the form

$$y_m^+ + 2l^+ = \delta^+. \tag{17}$$

Thus, in the first approximation with internal turbulent flow for $y^+ \geqslant y_m^+$

$$\nu^+ = \nu^+(y_m^+). \tag{18}$$

In Fig. 3 the theoretical dependences (16) and (18) are being compared with the experimental data of [10] whose authors studied fully developed flow in channels. There is satisfactory correlation between the experimental data and expressions (16) and (18).

The authors of [2-4, 6, 9] present the limits of applicability and different variants of the derivation of the universal distribution of the mean longitudinal speed in the turbulent core of a turbulent boundary layer

$$U = R. \tag{19}$$

We will show that if we know the logarithmic speed distribution [1]

$$u^+ = A \ln y^+ + B, \tag{20}$$

relation (19) can also be obtained on the basis of the following considerations.

It was shown in [18] that when the simplest flow conditions obtain, A may change from 2.1 to 3.0, according to published experimental data. The scatter of the values of B is even greater. According to the data of [11], $B \approx 4.0$-6.0. The boundary of (20) is not determined either. Usually the range of applicability of the logarithmic law is taken to extend from the boundary of Karman's transition region of $y \approx 0.1\delta$. On the other hand, a modification of (20) in the form of a law of defect of the speed

$$u_\delta^+ - u^+ = A \ln\left(\frac{\delta}{y}\right) + B_1 \tag{21}$$

is extended to the zone lying between the boundary of the transition region and $y = \delta$. In [11] it was pointed out that in consequence of the small value of the defect of speed with $y/\delta \sim 1$ it is extremely difficult to determine B_1 experimentally. According to a very rough approximation in [18], for flow in a circular pipe and in a flat channel $B_1 \approx 0.4$–0.8, for flow on a plate $B_1 \approx 2.4$. There are different views on how to explain the scatter of the values of A, B, B_1. According to [11], $A = A(Re)$, $B = B(Re)$, $B_1 = B_1(Re)$. In the review [18] the scatter of the values of the constants is explained by experimental errors, both in direct measurement and on account of excluding additional parameters affecting the functions A, B, B_1.

Since the values of A, B and the limits of (20) are not known with sufficient accuracy, we will assume that (20) is determinate in some range

$$0 < \delta_0(x) \leqslant y \leqslant \delta(x),\ 0 \leqslant x_1 \leqslant x \leqslant x_2, \tag{22}$$

where $\delta_0(x)$, $\delta(x)$ are boundary functions: the upper and lower boundaries of the turbulent core; in particular, $\delta(x)$ is the thickness of the turbulent boundary layer. For the mean longitudinal speed $u(x, y)$: $u_\delta(x) = u(x, \delta(x))$, $u_0(x) = u(x, \delta_0(x))$. Then we obtain from (20) directly

$$\frac{u - u_0}{u_\delta - u_0} = \frac{\ln(y/\delta_0)}{\ln(\delta/\delta_0)}. \tag{23}$$

Expression (23) has the important property that it does not depend on the replacement of scale δ_0 by some arbitrary y belonging to (22). In this sense all the y sets (22) are equivalent scales. In this sense all the y sets (22) are equivalent scales. In (20), y is the distance from the smooth wall. When the system of counting is shifted along the x axis coinciding with the wall, (23) does not change. However, (23) is not invariant to the shifting of the system of counting along y. Since in the general case turbulent flow may occur not only on a smooth wall but also, e.g., on a rough wall, or in other cases without a wall, all the y from (22) are physically not distances from the wall but turbulent scales (or distances from an unreal (pseudosmooth) wall), provided the values of y ensure equality (23).

We introduce the hypothesis that there exists a class of liquids for which with zero pressure gradient all y belonging to (22) and satisfying (23) are in the above-explained sense equivalent to the viscous scale $l_*(x) = y$ for $u = u_*(x)$; in particular, let $l_*(x) = \nu/u_*(x)$. Then by substituting l_* and u_* for δ_0 and u_0 we obtain immediately from (23) that

$$\frac{u^+ - 1}{u_\delta^+ - 1} = \frac{\ln y^+}{\ln \delta^+} \tag{24}$$

or

$$u^+ = \frac{1}{\Psi_\delta} \ln y^+ + 1, \tag{25}$$

where $\Psi_\delta = \ln\delta^+/(u_\delta^+ - 1)$. The obtained relations (24), (25) are identical with (19). It was shown in [2] that (19), (24), (25) are determinate in the range R [0.7; 1], which corresponds to the approximate range of change of y/δ from 0.05 to 1, i.e., to the turbulent core.

It was pointed out in [2-9] that equality (25) differs from (20) by the fact that it is physically real, i.e., it corresponds with sufficient accuracy to the known experimental data, does not contain empirical constants, describes a broad range of near-wall flows including the effect of mass force fields nonisothermy, and other factors on the turbulent boundary layer, and greately simplifies engineering calculations of the averaged characteristics of the turbulent boundary layer.

The authors of [3, 4, 8] present universal relations of molar heat and mass transfer. According to the model of superposition of vortices [3] whose rolling is examined in the proper system of counting, we can on the basis of the above-mentioned relations (taking into account that over the surface, on which the averaged vortex with radius l rolls, the flux density τ of the impulse, the heat flux density q, the mass flux density j are transferred while the averaged vortex itself is characterized by the rolling speed u_l, and also by the differences in temperature ϑ_l and concentration c_l) obtain:

$$\tau = \rho u_l u_l, \tag{26}$$

$$q\,\mathrm{Pr}_t = \rho u_l c_p \vartheta_l, \tag{27}$$

$$j\,\mathrm{Sc}_t = \rho u_l c_l. \tag{28}$$

Expressions (26)-(28) show the analogy between molar transfer of momentum, heat, and mass in the local base of the turbulent vortex. Their respective right-hand sides represent the flux densities of the impulse, heat, and mass, transferred by the averaged vortices through rolling, whereas the left-hand sides are the true densities of the fluxes damped to No. 1, the turbulent Prandtl number Pr_t, and the turbulent Schmidt number Sc_t.

Thus the model of [2-9] made it possible to unify the molar stress and viscosity profiles and some other relations correlating the averaged characteristic of the turbulent core of the turbulent boundary layer.

NOTATION

u, mean longitudinal speed, m/sec; ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; τ, tangential stress, N/m^2; $u_* = \sqrt{\tau_W/\rho}$, dynamic speed, m/sec; δ, thickness of the boundary layer, m; δ_0, thickness of the laminary sublayer, m; $y^+ = yu_*/\nu$, dimensionless coordinate; $u = u/u_*$, dimensionless speed; $R = \ln y^+/\ln\delta^+$, generalized dimensionless distance from the wall; $U = (u^+ - 1)/(u_\delta^+ - 1)$, generalized dimensionless speed; $\Psi = \ln y^+/(u^+ - 1)$, criterial function of the turbulent boundary layer; $\tau^+ = \tau/\tau_W$, $\sigma_i^+ = \sigma_i/u_*$, dimensionless terms of the stress tensor; $\nu^+ = \nu_t/\nu$, dimensionless coefficient of viscosity; Pr_t, turbulent Prandtl number; Sc_t, turbulent Schmidt number. Subscripts: *, δ, 0, flow parameters for $y^+ = 1$, $y^+ = \delta^+$, and $y^+ = \delta_0^+$, respectively; W, wall parameter; u, v, w, concerns longitudinal speed, and transverse speeds perpendicular and parallel to the wall, respectively; t, parameter of the turbulent core; l, parameter of the averaged vortex; k, internal flow.

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